DEVELOPING NUMBER SENSE

Kerri Smith, Fish Creek and District Primary School

Figure 1. These students are trying to create a loop of dominoes that have different representations of numbers to 6.

WHAT IS NUMBER SENSE?

> '...good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualising them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.'
> - Hilde Howden

Thomas Carpenter talks about students progressing from direct modelling to using counting-on to solve simple number problems. Many teachers then try to move them straight into memorising facts, and while some students can do this, many cannot.

Students who do not automatically know their facts resort back to counting one by one on their fingers. We need to help these students figure out facts when they do not automatically know them by working on derived strategies.

The ultimate goal of building number sense is to promote deeper thinking and not just knowledge of how to calculate answers.

To achieve this, John Van de Walle (et al.) describes how teachers in the junior years need to provide many experiences that can help students understand the essential concepts that make up good number sense.

These are:

- spatial relationships
- knowing one/two more, and one/two less
- benchmarking to five and ten
- part/part/whole relationships

SPATIAL RELATIONSHIPS

The term spatial relationships refers to the ability to recognise how many items in a collection without counting one by one (subitising) and building connections so that students know how one number relates to another.

If students can visualise the quantity of a number, as opposed to the digits, they can more readily recognise part/part/whole relationships. We want them to see that five dots can also be seen as four and one, three and two, two and two and one, etc. Being able to subitise enables students to use counting-on and counting-back more effectively.

One of the best ways to do this is to use a rekenrek (also called a number rack). This useful tool comes in 20 or 100 beads.

Figure 2. Rekenreks are really useful for teaching students to visualise quantities and to see how one number relates to another.

Beginning students can easily make a ten bead rekenrek by threading beads onto a pipe cleaner and attaching to cardboard. The beads are arranged in rows of ten so that kids can instantly recognise five and ten.

Students can see how the teens and multiples of ten are made up of groups of ten and 'some more'. They can also see that 90 is a lot more than 19, but 45 and 50 are similar quantities. When asked to show 73, students will soon realise that it is more than 50 and they will build up and use their knowledge of place value to slide 50 across then two more tens and three loose ones.

Addition and subtraction become visual as they physically work with the beads to add groups or to find the difference between two quantities.

Number paths also allow students to see the difference in magnitude of smaller numbers. When I ask my Foundation students to show me what eight looks like, I want to see the quantity, not the numeral. By sliding the counters onto the number path, they can see that eight also

Figure 3. Number paths (as opposed to number lines) can also help them to see quantities and relationships.

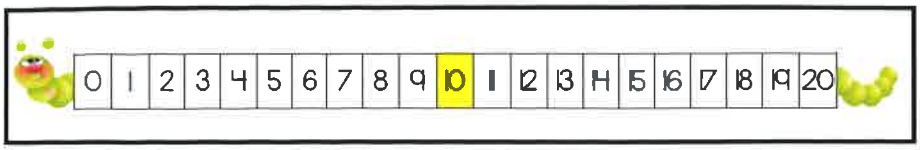

Figure 4. Games such as Heads and Tails help students to see how numbers relate to benchmarks.

includes all the numbers less than eight. They can easily see what is one and two more or less than eight; they can see that eight is nearly ten, etc. Add a magnetic frog and students can solve simple addition and subtraction problems by jumping it backwards and forwards.

Any work students do using ten frames helps to build their awareness of spatial relationships.

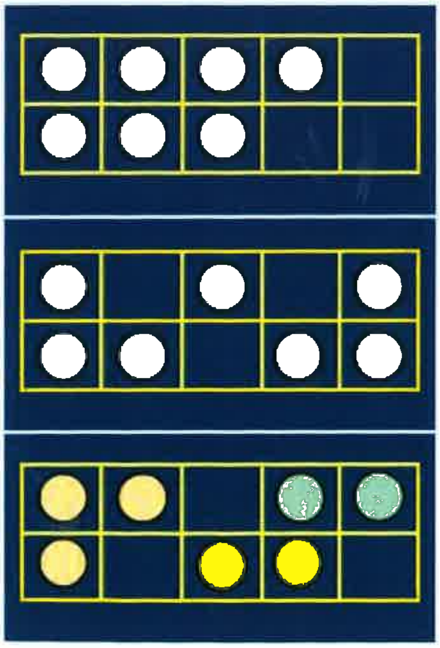

Figure 5. Ten frames are useful for visualising numbers, comparing them and breaking them into smaller parts.

We need to ensure students work with numbers in different formations on the ten frame and not just use a 'fill the top row first' configuration. Sometimes they will fill them 'pair-wise', with one on the top and one underneath and then go left to right. This configuration will also enable them to see why seven is an odd number because there is one counter without a partner.

When counters are randomly placed on a ten frame, students can start to see part/part/whole relationships, such as seven is three and four, it is also three and two and two, it can be three and one and three, etc.

ONE/TWO MORE AND LESS

Students with good number sense automatically know what one or two more or less than any number is without having to work it out. This ability aids their mental computation and can speed up their processing to solve problems. As Albert, a foundation student, explained…28 + 7 is easy 'because I can just take two off the seven and that gives me 30, then I just have to add five, so it's 35.' Knowing one and two more and less than seven and eight enabled Albert to solve not only 28 + 7, but he could also apply this to work out what 108 + 7 is. Students who have this understanding can also solve problems such as 59 + 25 by seeing it as 60 + 25, less one.

One way to help students understand this concept is to play *Heads and Tails*. This game is played in pairs, with one placing a counter above ten and one below it. Using dice with +1, +2, –1, –2 and a double-headed arrow on the remaining two sides, they take turns to roll the dice and work out what one or two more or less than the number they are on is and move their counter. If they roll the arrow, they swap places with each other and take over the other's counter. The aim is to be the first player to land on either zero or 20.

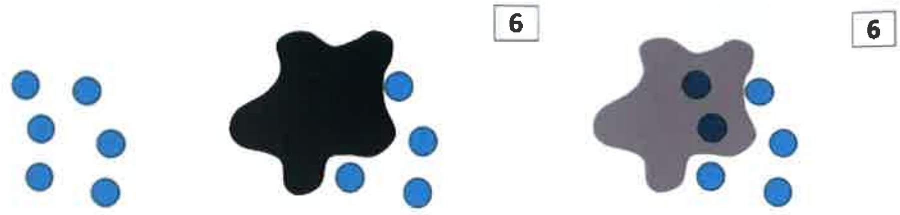

Figure 7. Steve Wyborney's SPLAT problems are great for showing part/part/whole relationships.

BENCHMARKING TO 5 AND 10

When students can relate numbers to five and ten, such as knowing that seven is more than five, but not more than ten, they can carry out more efficient problem solving because it helps them to understand place value. It also aids their mental computation because they can 'see' the number quantities in their heads. 'Make-a-ten' is a useful strategy for counting-on, but if kids don't know a number's relationship to ten, they can't access this method. Students need to develop the concept of 'five-ness' before they can recognise 'ten-ness'. Once they have a grasp of recognising and counting in tens, they can use the visual appearance of 50 on a rekenrek to benchmark larger numbers.

Figure 6. Bump.

Games such as 'BUMP' help students to benchmark numbers to five or ten, as can experiences that involve ten frames or Rekenreks.

PART/PART/WHOLE RELATIONSHIPS

Students need to see that numbers can be made up of two or more parts, and they can be combined to make larger groups. The three previous number sense concepts help build the idea that the number seven is not just seven items; it is also four and three, it is one less than eight, three less than ten and half of 14. As their number sense grows, students will come to see its usefulness as they work with multi-digit numbers, knowing that 87 can be 80 + 7, or 70 + 17, 13 less than 100, etc.

Steve Wyborney's SPLAT slideshows are an excellent way for kids to practise the part/part/whole relationship as they work out how many dots have been hidden by the SPLAT.

WHEN STUDENTS LACK NUMBER SENSE

Students in Years 3-6 who struggle often do so because they lack understanding of these four relationships. They do not see the connections between numbers and end up over-relying on algorithms that they do not understand or know how to use with accuracy. Once kids have been taught an algorithm, they tend to stop using mental strategies (Christina Tondevold) even though the latter is often far more efficient. Working out the difference between 1004 and 698 should not require an algorithm!

Too often, we see kids performing calculations without thinking through what is being asked of them, and so they fail to understand when their answers are unreasonable. The more we can get students decomposing numbers in fun and meaningful ways, the more proficient they will become with mental calculations.

Without any assistance at all, this group of foundation students worked together for 20 minutes to:

- determine the relationship between the placement of the numbers four, 61, 70 and 96 on a hundreds board
- decide which way the counting sequence would need to be placed
- place the remaining numbers on the board

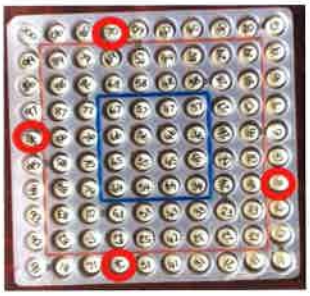

Figure 8.

(Note: The board was deliberately set up so that one would be placed in the bottom right-hand corner, counting upwards.)

The dialogue that they engaged in, coupled with their collective number sense lead them to success.

- 'Who put 86 there? It can't go there because all the sixes are down here!'
- 'Oh, look, it starts at the bottom and goes up, see? Here's one and it goes up to ten at the top, then it starts again down the bottom.'
- 'And look, all the threes go across this way!'
- 'How long did it take us? Did we break the world record?'

REFERENCES AND RESOURCES

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AUTHOR

Kerri Smith started teaching at Neerim South and has spent all of her career in Gippsland. She has been teaching at Fish Creek Primary School for the past 13 years. She wanted to be a teacher ever since Year 6, when she was asked to look after the preps for the afternoon when their teacher went home sick. Kerri even got to write on the blackboard that day (which students were never allowed to do!) and that's the day she decided to take up teaching. She has never wavered from that decision and still loves the works she does.

Email: smith.kerri.e@edumail.vic.gov.au

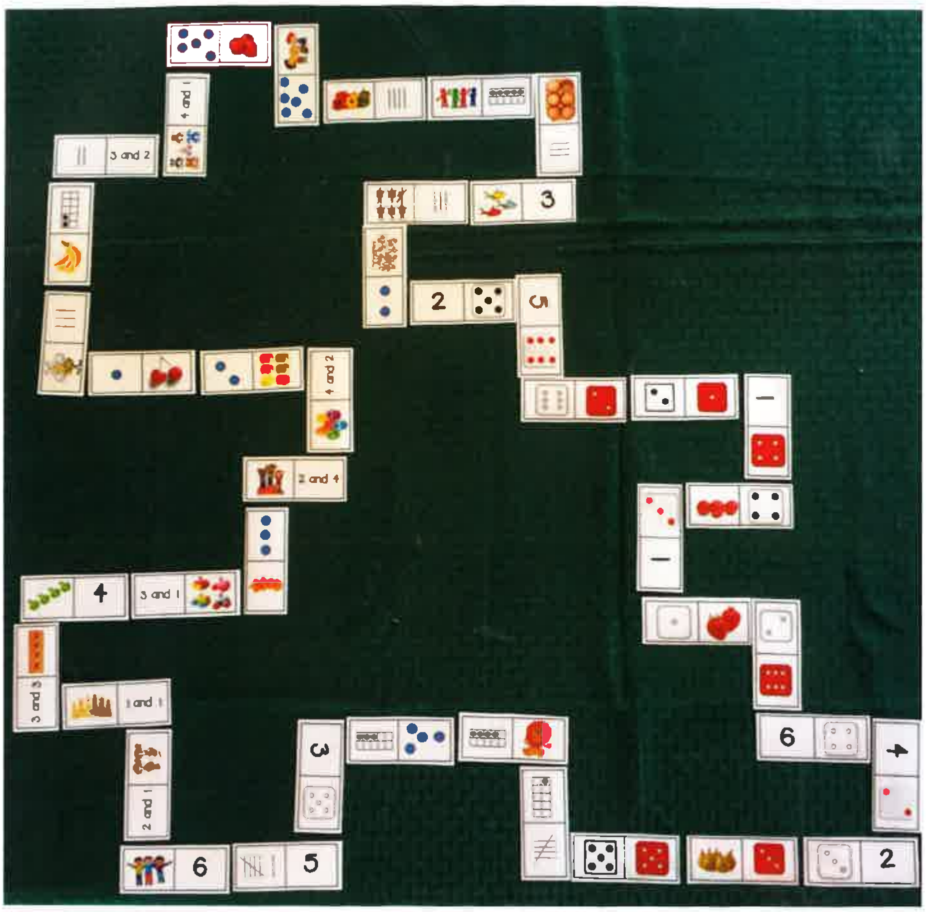

Figure 9. A completed domino loop.

Figure 10. Prep students collaborating to solve the problem.